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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2018
SECOND YEAR [BATCH 2017-20]
MATHEMATICS FOR ECONOMICS [General]
Paper : III

Date : 22/12/2018
Time : 11 am – 2 pm
Full Marks : 75

[Use a separate Answer Book <u>for each Group</u>]

<u>Group – A</u>

Answer <u>any four</u> questions from <u>Question nos. 1 to 6</u> : [4×5]

1. For what values of the number r is the function $f(x,y)=\begin{cases}\frac{(x+y)^r}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ continuous on $\mathbb{R}^2$? [5]

2. (a) Let *u* be a homogeneous function of x and y of degree n. Then prove that
$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$$ [3]
(b) State Schwarz's theorem. [2]

3. Suppose that f is a differentiable function of one variable. Show that all the tangent planes to the surface $z=xf\left(y/x\right)$ intersect at a common point. [5]

4. (a) For the improper integral $\int_{-1}^{1}\frac{dx}{x^3}$ show that Cauchy principal value exist but the integral does not exist. [3]
(b) Show that $B(m,n)=2\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\ \cos^{2n-1}\theta\, d\theta\ (m,n>0)$. [2]

5. If f(x,y,z) = x sin(yz), (a) find the gradient of f and (b) find the directional derivative of f at (1,3,0) in the direction of $\vec{v}=\hat{i}+2\hat{j}-\hat{k}$. [5]

6. (a) If $u=f\left(x^2+2yz, y^2+2zx\right)$, then prove that
$$\left(y^2-zx\right)\frac{\partial u}{\partial x}+\left(x^2-yz\right)\frac{\partial u}{\partial y}+\left(z^2-xy\right)\frac{\partial u}{\partial z}=0.$$ [3]
(b) Show that the improper integral $\int_2^{\infty}\frac{dx}{x\log x}$ diverges to ∞. [2]

Answer <u>any one</u> question from <u>Question nos. 7 and 8</u> : [1×10]

7. (a) The roots of the equation in λ, $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ are u,v,w.
Prove that the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}=-2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$ [5]
(b) Use the Lagrange multiplier method to prove that the triangle with maximum area with a given perimeter *p* is equilateral. [5]

8. (a) Find $\lim_{n\to\infty}\left\{\left(1+\frac{1}{n^2}\right)^{\frac{2}{n^2}}\left(1+\frac{2^2}{n^2}\right)^{\frac{4}{n^2}}\left(1+\frac{3^2}{n^2}\right)^{\frac{6}{n^2}}\cdots\cdots\left(1+\frac{n^2}{n^2}\right)^{\frac{2n}{n^2}}\right\}$. [4]

(b) Find the range of the values of x for which the function $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Also find the point of inflexion if any. [6]

Answer <u>any one</u> question from <u>Question nos. 9 and 10</u> : [1×10]

9. (a) Suppose the Hicksian demand functions of an expenditure-minimizing consumer are given by $h_1(p_1,p_2,u)=\frac{u}{2}\sqrt{\frac{p_2}{p_1}}$ and $h_2(p_1,p_2,u)=\frac{u}{2}\sqrt{\frac{p_1}{p_2}}$. Find the equation of the indirect utility function. [4]

(b) Show that an isoquant corresponding to the production function $Q=\text{Max}\left\{L+K,\frac{4LK}{L+K}\right\}$ is convex, but not strictly-convex. [6]

10. (a) Suppose that the expenditure function of an expenditure-minimizing consumer is given by

$$e(p_1,p_2,u)=\frac{up_1p_2}{p_1+p_2}$$

Find the Marshallian demand function for commodity 1. [4]

(b) Consider the production function F(L,K) = Min[2L+3K, L+4K] for L,K ≥ 0 ,where L and K are the amounts of Labour and Capital respectively.

i) Draw the isoquant for output =10.
ii) Show that the production function exhibits CRS.
iii) Find the cost-minimizing choice of the firm when w = r = 1.
iv) Find the expansion path. [2+1+2+1]

<u>Group – B</u>

Answer <u>any seven</u> questions from <u>Question Nos. 11 to 21</u> **:** [7×5]

11. Consider the following C-D production function $Q = f(K,L) = K^\alpha L^\beta$, $\alpha+\beta = 1$. Suppose L grows at a fixed rate i.e. $\frac{dL}{dt}=\lambda L.$ what is saved is invested and there is no depreciation, so that $\frac{dK}{dt}=s.Q.$ Define $k=\frac{K}{L}$ be the capital per unit of labour. Calculate the rate of change of k with respect to time. Hence construct the fundamental differential equation of the model as, $\dot{k}+\lambda k=sk^\alpha$, where $\dot{k}=\frac{dk}{dt}$. [5]

12. (a) State Clairaut's form of differential equation of first order and higher degree. What do you mean by singular solution of a differential equation? [1+2]

(b) Find the singular solution of the differential equation $y=px+p^2$, where $p=\frac{dy}{dx}$. [2]

13. Solve the differential equation $\frac{dy}{dx}+ax=sx^{\alpha}$. [5]

14. (a) Define integrating factor (I.F.) of a differential equation. [1]

(b) Examine whether the equation

$$xy\,dx+\left(2x^2+3y^2-20\right)dy=0$$

is exact or not. Hence solve the equation. [1+3]

15. Solve $xy\frac{dy}{dx}-y^2=\left(x+y\right)^2 e^{-\frac{y}{x}}$ [5]

16. Solve $\frac{d^2y}{dx^2}+4y=e^{3x}$ [5]

17. Find the solution of the initial value problem $x^2\frac{dy}{dx}+xy=1, \quad x>0, \quad y(1)=2$. [5]

18. Using the method of undetermined coefficients, find the general solution of

$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=3xe^x$ [5]

19. Solve the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=2e^x$ and find the particular solution if y =3 when x = 0 and y =8 when $x=\log_e 2$ [3+2]

20. Apply the method of variation of parameters to solve the differential equation [5]

$$\frac{d^2y}{dx^2}+9y=\sec 3x$$

21. Transform the single linear differential equation $\frac{d^3x}{dt^3}+t\frac{d^2x}{dt^2}+2t^3\frac{dx}{dt}-5t^4=0$ into a set of first order differential equation. [5]

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